sagen mit Hilfe der angegebenen Besetzungszahlen und Diagramme nur auf allgemeine Zusammenhänge hinweisen kann. Eine quantitative Auswertung aller Ergebnisse ist dabei nicht zu erwarten. Aber schon qualitative Angaben können die Gründe für das Zustandekommen vieler Effekte klären, eine Ordnung der Erfahrungen herbeiführen und gewisse Voraussagen ermöglichen.

[7] Man vergleiche hier besonders die Analyse der Elektronenverteilungen nach R. S. MULLIKEN, J. Chem. Phys. **23**, 1833, 1841, 2338, 2343 [1955], mit der die hier angegebene in wesentlichen Zügen übereinstimmt.

[8] Man vergleiche Arbeitsbericht Nr. 11 der Arbeitsgruppe Quantenchemie (Max-Planck-Institut für Physik und Astrophysik, München, H. PREUSS, Zur Interpretation der Ergebnisse des SCF-MO-P (LCGO)-Verfahrens, S. 54.

A Finite-Amplitude Solution of the Self-Consistent Vlasov Equations

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The paper deals with the propagation of a finite-amplitude circularly polarized electromagnetic plane wave through a fully ionized hot collisionless plasma, composed of Q ($\geqq 1$) types of ions and of electrons, as a strict solution of the non-relativistic self-consistent Vlasov equations within the scope of some given applicability criterions. A constant magnetic field and a constant total plasma stream, both parallel to the wave propagation direction, are allowed. Wave propagation and particle motion are separately discussed. Particularly, the possible existence of an anomalous wave, propagating with light speed through multi-component ($Q > 1$) plasmas with arbitrary ratios plasmafrequency/wavefrequency, and the possible existence of multi-component plasmas with temperatures, far below thermonuclear temperatures, but under electromagnetically generated fusion conditions, are proved. The treated solution is of physical and astrophysical interest, likewise.

1. Introduction

It is a well-known fact [1], that a finite-amplitude circularly polarized electromagnetic plane wave can propagate through a fully ionized cold plasma, composed of Q ($\geqq 1$) types of ions and of electrons.

This paper is dealing with the question of a possible finite-amplitude circularly polarized electromagnetic plane wave propagation through a fully ionized hot plasma, composed of Q ($\geqq 1$) types of ions and of electrons, as a strict solution of the non-relativistic self-consistent Vlasov equations. The answer, in no way evident a priori, is a positive one, if certain applicability criterions are fulfilled.

In Sect. 3, the basic equations are solved. In Sect. 4, the applicability criterions for the solution are derived. Section 5 gives a discussion about the two aspects of the considered solution: wave propagation and particle motion. Particularly, the case of multi-component ($Q>1$) plasmas leads to conclusions of physical interest.

For additional research on transverse wave propagation in hot plasmas refer to [2].

2. The Basic Equations

The self-consistent Vlasov equations are given by the following set of Vlasov, Maxwell, and moment equations for a fully ionized plasma, composed of Q types of ions and of electrons (the volt-amp-cm-sec-system of units has been used).

$$\left[\frac{\partial}{\partial t} + \boldsymbol{w}\cdot\nabla_r + \frac{Z_q\, e}{m_q}\,(\boldsymbol{E} + (\boldsymbol{w}\times\boldsymbol{B}))\cdot\nabla_w\right] \cdot f_q(\boldsymbol{r},\boldsymbol{w},t) = 0; \qquad (2.1)$$

$$\left[\frac{\partial}{\partial t} + \boldsymbol{w}\cdot\nabla_r - \frac{e}{m_-}\,(\boldsymbol{E} + (\boldsymbol{w}\times\boldsymbol{B}))\cdot\nabla_w\right] \cdot f_-(\boldsymbol{r},\boldsymbol{w},t) = 0; \qquad (2.2)$$

$$\nabla\times\boldsymbol{E} = -\,\partial\boldsymbol{B}/\partial t\,, \quad \nabla\times\boldsymbol{H} = \boldsymbol{j} + \partial\boldsymbol{D}/\partial t; \qquad (2.3)$$

$$\nabla\cdot\boldsymbol{D} = \lambda\,, \quad \nabla\cdot\boldsymbol{B} = 0; \qquad (2.4)$$

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[1] W. LÜNOW, Plasma Physics **10**, 888 [1968].

[2] H. S. C. WANG and M. S. LOJKO, Physics Fluids **6**, 1458 [1963]. — J. J. GIBBONS and R. E. HARTLE, Physics Fluids **10**, 189 [1967]. — B. ABRAHAM-SHRAUNER, Physics Fluids **11**, 1162 [1968]. — H. C. HSIEH, Physics Fluids **11**, 1497 [1968]. — J. A. WEBER and W. A. GUSTAFSON, Plasma Physics **9**, 713 [1967].

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$$\boldsymbol{D}=\varepsilon_0 \boldsymbol{E}, \quad \boldsymbol{B}=\mu_0 \boldsymbol{H}, \quad c^2=(\varepsilon_0 \mu_0)^{-1}; \tag{2.5}$$

$$\lambda=-e n_- + e \sum n_q Z_q, \tag{2.6}$$

$$\boldsymbol{j}=-e n_- \boldsymbol{v}_- + e \sum n_q Z_q \boldsymbol{v}_q, \quad q=1,2,\dots,Q; \tag{2.7}$$

$$n_q=\iiint f_q(\boldsymbol{r},\boldsymbol{w},t)\,\mathrm{d}\boldsymbol{w}; \tag{2.8}$$

$$n_-=\iiint f_-(\boldsymbol{r},\boldsymbol{w},t)\,\mathrm{d}\boldsymbol{w}; \tag{2.9}$$

$$n_q \boldsymbol{v}_q=\iiint \boldsymbol{w} f_q(\boldsymbol{r},\boldsymbol{w},t)\,\mathrm{d}\boldsymbol{w}; \tag{2.10}$$

$$n_- \boldsymbol{v}_-=\iiint \boldsymbol{w} f_-(\boldsymbol{r},\boldsymbol{w},t)\,\mathrm{d}\boldsymbol{w}. \tag{2.11}$$

3. The Solution

3.1. First Assumption

The distribution functions $f_q(\boldsymbol{r},\boldsymbol{w},t)$ and $f_-(\boldsymbol{r},\boldsymbol{w},t)$ are assumed to have the general forms

$$f_q(\boldsymbol{r},\boldsymbol{w},t)=n_q\left[\frac{m_q}{2\pi K T}\right]^{3/2} \cdot \exp\left[-\frac{m_q}{2KT}(\boldsymbol{w}-\boldsymbol{v}_q(\boldsymbol{r},t))^2\right], \tag{3.1}$$

$$f_-(\boldsymbol{r},\boldsymbol{w},t)=n_-\left[\frac{m_-}{2\pi K T}\right]^{3/2} \cdot \exp\left[-\frac{m_-}{2KT}(\boldsymbol{w}-\boldsymbol{v}_-(\boldsymbol{r},t))^2\right]. \tag{3.2}$$

Eqs. (3.1) and (3.2) imply space-time dependent drift-velocities $\boldsymbol{v}_q$ and $\boldsymbol{v}_-$, constant particle densities n_q and n_-, and a constant thermal equilibrium temperature T of the plasma. K is the Planck-Boltzmann constant.

Introducing Eq. (3.1) into Eq. (2.1) and Eq. (3.2) into Eq. (2.2) it follows, respectively, that

$$(\boldsymbol{w}-\boldsymbol{v}_q)\cdot\left[\frac{\partial \boldsymbol{v}_q}{\partial t}+(\boldsymbol{w}\cdot\nabla_r)\,\boldsymbol{v}_q - \frac{Z_q e}{m_q}(\boldsymbol{E}+(\boldsymbol{w}\times\boldsymbol{B}))\right]=0, \tag{3.3}$$

$$(\boldsymbol{w}-\boldsymbol{v}_-)\cdot\left[\frac{\partial \boldsymbol{v}_-}{\partial t}+(\boldsymbol{w}\cdot\nabla_r)\,\boldsymbol{v}_- + \frac{e}{m_-}(\boldsymbol{E}+(\boldsymbol{w}\times\boldsymbol{B}))\right]=0. \tag{3.4}$$

The thermal velocities of the particles are given by

$$\boldsymbol{s}_q=\boldsymbol{w}-\boldsymbol{v}_q \quad \text{and} \quad \boldsymbol{s}_-=\boldsymbol{w}-\boldsymbol{v}_-. \tag{3.5}$$

For three different vectors $\boldsymbol{a}$, $\boldsymbol{b}$, and $\boldsymbol{c}$ it holds that

$$(\boldsymbol{a}-\boldsymbol{b})\cdot(\boldsymbol{a}\times\boldsymbol{c})=(\boldsymbol{a}-\boldsymbol{b})\cdot(\boldsymbol{b}\times\boldsymbol{c}). \tag{3.6}$$

With Eqs. (3.3) to (3.6) one obtains that

$$\boldsymbol{s}_q\cdot\left[\frac{\partial \boldsymbol{v}_q}{\partial t}+(\boldsymbol{s}_q\cdot\nabla_r)\,\boldsymbol{v}_q+(\boldsymbol{v}_q\cdot\nabla_r)\,\boldsymbol{v}_q - \frac{Z_q e}{m_q}(\boldsymbol{E}+(\boldsymbol{v}_q\times\boldsymbol{B}))\right]=0; \tag{3.7}$$

$$\boldsymbol{s}_-\cdot\left[\frac{\partial \boldsymbol{v}_-}{\partial t}+(\boldsymbol{s}_-\cdot\nabla_r)\,\boldsymbol{v}_-+(\boldsymbol{v}_-\cdot\nabla_r)\,\boldsymbol{v}_- + \frac{e}{m_-}(\boldsymbol{E}+(\boldsymbol{v}_-\times\boldsymbol{B}))\right]=0. \tag{3.8}$$

According to Eq. (3.5), $\boldsymbol{s}_q$ and $\boldsymbol{s}_-$ contain the independent variable $\boldsymbol{w}$. Eqs. (3.7) and (3.8) must be valid for arbitrary vectors $\boldsymbol{s}_q$ and $\boldsymbol{s}_-$. Therefore, it follows that

$$\frac{\partial \boldsymbol{v}_q}{\partial t}+(\boldsymbol{s}_q\cdot\nabla_r)\,\boldsymbol{v}_q+(\boldsymbol{v}_q\cdot\nabla_r)\,\boldsymbol{v}_q - \frac{Z_q e}{m_q}(\boldsymbol{E}+(\boldsymbol{v}_q\times\boldsymbol{B}))=0, \tag{3.9}$$

$$\frac{\partial \boldsymbol{v}_-}{\partial t}+(\boldsymbol{s}_-\cdot\nabla_r)\,\boldsymbol{v}_-+(\boldsymbol{v}_-\cdot\nabla_r)\,\boldsymbol{v}_- + \frac{e}{m_-}(\boldsymbol{E}+(\boldsymbol{v}_-\times\boldsymbol{B}))=0. \tag{3.10}$$

With the first assumption, contained in Eqs. (3.1) and (3.2), the basic Eqs. (2.1) and (2.2) can be replaced by Eqs. (3.9) and (3.10), respectively.

Eqs. (2.8) to (2.11) are fulfilled by the distribution functions Eqs. (3.1) and (3.2).

Now it is sufficient to consider Eqs. (3.9), (3.10), and (2.3) to (2.7), only.

3.2. Second Assumption

The equation set Eqs. (3.9), (3.10), and (2.3) to (2.7) is assumed to have the following solution (Cartesian coordinates)

$$\begin{aligned}
\boldsymbol{v}_q(v_{qx},v_{qy},v_{qz}) &= \boldsymbol{v}_q(v_0,v_{qy},v_{qz}),\\
\boldsymbol{v}_-(v_{-x},v_{-y},v_{-z}) &= \boldsymbol{v}_-(v_0,v_{-y},v_{-z}),\\
\boldsymbol{E}(E_x,E_y,E_z) &= \boldsymbol{E}(0,E_y,E_z),\\
\boldsymbol{B}(B_x,B_y,B_z) &= \boldsymbol{B}(B_0,B_y,B_z),
\end{aligned} \tag{3.11}$$

$$\begin{aligned}
v_{qy} &= v_q \sin\xi, & v_{qz} &= \pm v_q \cos\xi,\\
v_{-y} &= v_- \sin\xi, & v_{-z} &= \pm v_- \cos\xi,\\
E_y &= -E\cos\xi, & E_z &= \pm E\sin\xi,\\
B_y &= \mp B\sin\xi, & B_z &= -B\cos\xi,\\
\xi &= k(x-ut)+\varphi = kx-\omega t+\varphi. & &
\end{aligned} \tag{3.12}$$

Eqs. (3.11) and (3.12) are representing a circularly polarized electromagnetic plane wave, propagating along the positive x-direction. The upper signs indicate right-hand polarization, the lower signs indicate left-hand polarization. v_0, B_0, and all maximum amplitudes v_q, v_-, E, and B are assumed to be constant with respect to x, $\boldsymbol{w}$, and time t.

The introduction of Eqs. (3.11) and (3.12) into Eqs. (3.9), (3.10), and (2.3) to (2.7) leads to the following complete set of relations, still necessary for the determination of all properties of the considered solution.

$$\left[\left(1 \pm \frac{Z_q e B_0}{\omega m_q} - \frac{v_0}{u}\right) - \frac{s_{qx}}{u}\right] v_q = \frac{Z_q e}{\omega m_q}\left(1 - \frac{v_0}{u}\right) E, \tag{3.13}$$

$$\left[\left(1 \mp \frac{e B_0}{\omega m_-} - \frac{v_0}{u}\right) - \frac{s_{-x}}{u}\right] v_- = -\frac{e}{\omega m_-}\left(1 - \frac{v_0}{u}\right) E, \tag{3.14}$$

$$B = \frac{1}{u} E, \tag{3.15}$$

$$n_- = \sum n_q Z_q, \tag{3.16}$$

$$\frac{\mu_0 e}{\omega}(n_- v_- - \sum n_q Z_q v_q) = \frac{1}{u} B - \frac{1}{c^2} E = \frac{c^2 - u^2}{c^2 u^2} E. \tag{3.17}$$

3.3. *Third Assumption*

The thermal velocity components s_{qx} and s_{-x} along the wave propagation direction may have arbitrary values. For uniqueness it is further assumed that

$$\left|1 \pm \frac{Z_q e B_0}{\omega m_q} - \frac{v_0}{u}\right| \gg \left|\frac{s_{qx}}{u}\right|, \tag{3.18}$$

$$\left|1 \mp \frac{e B_0}{\omega m_-} - \frac{v_0}{u}\right| \gg \left|\frac{s_{-x}}{u}\right|, \tag{3.19}$$

in Eqs. (3.13) and (3.14), respectively. In words, s_{qx} and s_{-x} of almost all particles of each type of particle must fulfill Eqs. (3.18) and (3.19).

3.4. *Résumé*

Now the complete solution can be given in the following way –

f_q and f_- by Eqs. (3.1) and (3.2).
$\boldsymbol{v}_q$, $\boldsymbol{v}_-$, $\mathbf{E}$, and $\mathbf{B}$ by Eqs. (3.11) and (3.12).

The relation between B and E by Eq. (3.15).

Using Eqs. (3.18) and (3.19) the relations between v_q and E and between v_- and E by

$$v_q = \frac{1 - v_0/u}{1 \pm M_q \Phi - v_0/u} \frac{M_q e}{\omega m_-} E, \tag{3.20}$$

$$v_- = -\frac{1 - v_0/u}{1 \mp \Phi - v_0/u} \frac{e}{\omega m_-} E \tag{3.21}$$

with the definitions

$$\Phi = e B_0/\omega m_- \quad \text{and} \quad M_q = Z_q m_-/m_q .$$

Using Eqs. (3.17), (3.20), and (3.21) the dispersion relation by

$$n^2 = \left(\frac{c}{u}\right)^2 = 1 - \Theta^2\left(1 - \frac{v_0}{u}\right) \cdot \left[\frac{1}{1 \mp \Phi - v_0/u} + \sum \frac{N_q M_q}{1 \pm M_q \Phi - v_0/u}\right] \tag{3.22}$$

with the definitions

$\Theta = \omega_p/\omega$, $\omega_p = \mu_0 e^2 c^2 n_-/m_-$, $N_q = Z_q n_q/n_-$,

and $\Sigma N_q = 1$ [by means of Eq. (3.16)].

4. The Applicability Criterions

One applicability criterion for the considered solution originates in the general theory of the Vlasov equations, saying that the number of particles n_{tot} in the Debye volume L_D^3 has to be very large against one. In the considered case it holds that[3]

$$n_{\text{tot}} = n_- + \sum n_q = \text{const}, \tag{4.1}$$

$$\frac{1}{L_D^2} = \frac{e^2}{\varepsilon_0 K T}(n_- + \sum n_q Z_q^2), \tag{4.2}$$

$$n_{\text{tot}} L_D^3 \gg 1 . \tag{4.3}$$

From Eqs. (4.1) to (4.3) one obtains the applicability criterion

$$\left[\frac{e^2}{\varepsilon_0 K} \frac{1 + \Sigma N_q Z_q}{(1 + \Sigma (N_q/Z_q))^{2/3}}\right]^{3/2} n_-^{\frac{1}{2}} \ll T^{3/2}. \tag{4.4}$$

Further applicability criterions are Eqs. (3.18) and (3.19). Two facts should be pointed out.

At first, the plasma temperature T is assumed to be isotropic. There is no preference of the wave propagation direction related to T. In Eqs. (3.18) and (3.19) s_{qx} and s_{-x} could stand for all three spatial coordinate directions.

At second, in Eqs. (3.18) and (3.19) s_{qx} and s_{-x} are not compared with the plasma-internal macroscopic particle speeds v_q and v_-, but, essentially, with the immaterial phase speed u of the wave. Generally, the plasma is allowed to be hot.

[3] D. C. Montgomery and D. A. Tidman, Plasma Kinetic Theory, McGraw-Hill Book Company, New York 1964.

Eqs. (3.18) and (3.19) do mean neither a "cold plasma in wave propagation direction, only" nor a "cold plasma, at all".

It is convenient to replace the components s_{qx} and s_{-x} of the thermal velocities in Eqs. (3.18) and (3.19) by the temperature T. With an isotropic T the equipartitition theorem says (related to the kinetic energy of translation along the x-axis) that

$$\sqrt{\overline{s_{qx}^2}} \cong \sqrt{\frac{K}{m_q} T} \quad \text{and} \quad \sqrt{\overline{s_{-x}^2}} \cong \sqrt{\frac{K}{m_-} T}\,. \quad (4.5)$$

The thermal velocity distributions along the three coordinate axes can be assumed as, essentially, one and the same. Therefore,

$$\overline{s_{qx}^2} \cong \overline{s_{qy}^2} \cong \overline{s_{qz}^2} \cong \tfrac{1}{3}\,\overline{(s_{qx}^2 + s_{qy}^2 + s_{qz}^2)} \quad \text{and}$$
$$\overline{s_{-x}^2} \cong \overline{s_{-y}^2} \cong \overline{s_{-z}^2} \cong \tfrac{1}{3}\,\overline{(s_{-x}^2 + s_{-y}^2 + s_{-z}^2)}\,.$$

In addition, the approximation will be used, that the root-mean-squares of s_{qx} and s_{-x} in Eq. (4.5) represent the individual speeds s_{qx} and s_{-x} sufficiently. This should be true, keeping in mind that the solution functions Eqs. (3.1) and (3.2) correspond to Maxwell-Boltzmann distributions. Then Eqs. (3.18) and (3.19) change into

$$\sqrt{T} \ll \sqrt{\frac{m_q}{K}}\, u \left| 1 \pm M_q \Phi - \frac{v_0}{u} \right|, \quad (4.6)$$

$$\sqrt{T} \ll \sqrt{\frac{m_-}{K}}\, u \left| 1 \mp \Phi - \frac{v_0}{u} \right|, \quad (4.7)$$

respectively.

The considered solution can be applied within a temperature range, defined by Eqs. (4.4), (4.6), (4.7) and dependent on the parameters of the plasma-wave-system.

An additional applicability criterion is the statement, that the absolute values of all material speeds as v_0, v_q, v_-, s_{qx}, and s_{-x} must be non-relativistic.

5. Discussion

The considered solution allows finite amplitudes for the particle and field quantities. The only restriction is, that the particle speeds have to be non-relativistic.

Eqs. (3.9) and (3.10) contain non-linear terms. The superposition principle is not valid.

Eqs. (4.6) and (4.7) are analogous to a relationship[4] between the phase speed of an electric (longitudinal) wave and the averaged thermal electron speed, parallel to the wave propagation direction, in an electron plasma. In a sense the given solution can be considered as a Landau-damping-free transverse one.

In spite of its very special form, the solution should be applicable to real plasma-wave-systems, if all necessary conditions are fulfilled within essential regions of (astrophysical or laboratory) plasmas under question.

From the physical standpoint just special cases make the given solution interesting, apart from the fact that it solves the self-consistent Vlasov equations strictly within the scope of the applicability criterions.

The further emphasis will be on special cases. It is useful to look on wave propagation and on particle motion separately.

5.1. The Wave Propagation

The wave propagation is governed by the dispersion relation Eq. (3.22).

Eq. (3.22) is a polynomial in n (or u) with the parameters N_q, M_q, Θ, Φ, and v_0. For physical solutions n must be real and non-negative.

The electromagnetic wave is a circularly polarized plane one, propagating along the positive x-direction with a phase speed u, given by Eq. (3.22) and with possible values equal to and greater or smaller than c. A streaming speed v_0 of the whole plasma along the wave propagation direction is allowed. u and v_0 are coupled by Eq. (3.22).

Assuming $\Theta \neq 0$ and $\Phi \neq 0$ in Eq. (3.22), one can approach to the special (limiting; v_0 non-relativistic!) case $n \to 1$ for $v_0 \to c$. v_0 must have the direction of u. Note that Θ may have arbitrary values, to a certain extent.

Assuming $\Theta \neq 0$ and $v_0 = 0$, one can obtain $n = 1$ ($u = c$) from the dispersion relation Eq. (3.22), if

$$\frac{1}{1 \mp \Phi} + \sum \frac{N_q M_q}{1 \pm M_q \Phi} = 0\,. \quad (5.1)$$

For a two-component plasma with electrons and one type of ion only, Eq. (5.1) has no physical solution with respect to Φ, on principle. In this case Eq. (5.1) results in $(Z_1 m_-)/m_1 = -1$. Z_1, m_1, and m_- are positive by their very natures, however.

Eq. (5.1) is physically solvable with respect to Φ, if, and only if, $Q > 1$ and two (or more) $M_q{}^s$ are different. With $Q = 2$, $N_1 \neq 0$, $N_2 \neq 0$, and $N_1 + N_2$

[4] L. D. Landau, Collected Papers, Pergamon Press, Oxford 1965, p. 451.

$=1$ it follows, for example, that

$$\Phi = \mp \frac{1 + N_1 M_1 + N_2 M_2}{M_1 M_2 + N_1 M_2 + N_2 M_1}. \tag{5.2}$$

Eq. (5.2) implies that essential quantities of both types of ions are in the plasma.

The dispersion relation Eq. (3.22) indicates the existence of a circularly polarized electromagnetic plane wave, running with light speed ($u = c$) through a hot collisionless multi-component ($Q > 1$) plasma of arbitrary Θ in a constant magnetic field B_0. This statement might have more significance for astrophysical than for laboratory multi-component plasmas of the treated kind.

In order to convey a certain idea about the wave propagation (and the particle motion, in section 5.2), one specialized example will be sketched in the following.

Taking the assumptions $\Theta \neq 0$, $v_0 = 0$, $n = 1$, $Q = 2$, $N_1 \neq 0$, $N_2 \neq 0$, $N_1 + N_2 = 1$, and Eq. (5.2) it follows from the applicability criterions Eqs. (4.4), (4.6), (4.7) that

$$\left[\frac{e^2}{\varepsilon_0 K} \frac{1 + N_1 Z_1 + N_2 Z_2}{(1 + (N_1/Z_1) + (N_2/Z_2))^{2/3}}\right]^{3/2} n_-^{\frac{1}{2}} \ll T^{3/2}, \tag{5.3}$$

$$\sqrt{T} \ll \sqrt{\frac{m_1}{K}}\, c\, \frac{N_1 \left| M_2 - M_1 \right|}{N_1 M_2 + N_2 M_1}, \tag{5.4}$$

$$\sqrt{T} \ll \sqrt{\frac{m_2}{K}}\, c\, \frac{N_2 \left| M_1 - M_2 \right|}{N_1 M_2 + N_2 M_1}, \tag{5.5}$$

$$\sqrt{T} \ll \sqrt{\frac{m_-}{K}}\, c\, \frac{1}{N_1 M_2 + N_2 M_1}, \tag{5.6}$$

respectively.

In Eqs. (5.4) to (5.6) the fact has been used that for all conceivable ions the M^s are in orders of magnitudes equal to or smaller than 10^{-3}. So terms, quadratic in M_1 and M_2, have been generally neglected against terms, linear in M_1 and M_2, and terms, linear in M_1 and M_2, have been generally neglected against one, keeping in mind that N_1, N_2, M_1, M_2 are positive numbers and that N_1 and N_2 have orders of magnitudes of one.

Now a further specialization will be made by choosing D-ions for the first type of ion (index 1) and T-ions for the second type of ion (index 2). In addition, $N_1 = N_2 = .5$ is assumed, what means that $n_1 = n_2 = .5\, n_-$. With the corresponding numerical values one obtains from Eqs. (5.3) to (5.6) that (n_- in cm^{-3}, T in °K)

$$(4.32 \times 10^{-3})\, n_-^{1/2} \ll T^{3/2}, \tag{5.7}$$

$$\sqrt{T} \ll 0.93 \times 10^6, \tag{5.8}$$

$$\sqrt{T} \ll 1.14 \times 10^6, \tag{5.9}$$

$$\sqrt{T} \ll 3.38 \times 10^8, \tag{5.10}$$

respectively.

Using Eqs. (5.7) to (5.10), for an electron density of $n_- = 10^{15}\ \mathrm{cm}^{-3}$ the solution can be applied within a temperature range of

$$10^5\ ^\circ\mathrm{K} \leqq T \leqq 10^7\ ^\circ\mathrm{K}, \tag{5.11}$$

keeping in mind that the root-mean-square of the thermal electron velocity is just still non-relativistic at 10^7 °K.

It should be pointed out that the temperature range Eq. (5.11) is valid for all non-relativistic macroscopic particle speeds. Particularly, the macroscopic particle speeds can be much smaller than the corresponding root-mean-squares of the thermal particle velocities. This means that the plasma is hot and that the small-amplitude macroscopic particle motion is not influenced by the high-amplitude thermal particle motion. Section 5.2 will show that macroscopic ion and electron speeds of one and the same order of magnitude are possible, in spite of the small electron-ion-mass ratios.

Generally, the given (non-relativistic) solution is no trivial extension of the well-known circularly polarized electromagnetic plane wave solution [1] in cold plasmas. In no way it is evident a priori that the solution in cold plasmas must automatically hold in hot plasmas, too.

One main result of this section is the possible wave propagation through plasma regions which are otherwise cut-off regions [5].

The case $u = c$ can be considered as a weak resonance one with $\Phi \cong \mp 10^3$, where $\left|1 \pm M_q \Phi\right|$ has an order of magnitude of one. For strong resonance cases, $\left|1 \pm M_q \Phi\right|$ or $\left|1 \mp \Phi\right|$ become much smaller than one and the upper limit in an equation like Eq. (5.11) is shifted downwards, normally. In weak resonance cases it is sufficient [5] to replace Eqs. (5.4) to (5.6) by $T \ll (m_-/K)\, u^2$.

5.2. The Particle Motion

Before getting back to the specialized example of Sect. 5.1, the general particle motion will be discussed with respect to the laboratory reference frame.

[5] W. LÜNOW, Physics Letters **29 A**, 214 [1969].

The electromagnetic field vectors of the wave are rotating about the wave propagation direction.

In each infinitesimal layer, perpendicular to the wave propagation direction ($x = \text{const}$), the macroscopic motion of each particle type, caused by the electromagnetic wave, corresponds to a parallel translation of the particle type entirety in the layer along a circle, also perpendicular to the wave propagation direction. All particle types in each infinitesimal layer have one and the same macroscopic velocity direction (apart from the signs) at each instant. There are phase shifts between the parallel translations in different layers, caused by the wave phase.

The characteristic parameter for both the rotation of the electromagnetic wave field and the parallel translation of the particle types is the angular frequency of the wave. The parallel translations have different circle radii and different orbital speeds along the translation circles for particle types with different M_q^s. The electromagnetic wave generates interpenetrating macroscopic streams of different particle types with well-defined streaming velocities, given by the components v_{qy}, v_{qz}, v_{-y}, and v_{-z}.

There is a thermal particle exchange between the layers, but the distribution functions Eq. (3.1) and (3.2) are conserved in a sense that the macroscopic velocities with the components v_{qy}, v_{qz}, v_{-y}, and v_{-z} are superposed on the thermal velocity distributions of the different particle types, only.

Returning now to the specialized example of Sect. 5.1, one obtains the following relations by means of Eqs. (3.20) and (3.21) (ω in sec^{-1}, E in V/cm)

$$|B_0| \cong 2.5 \times 10^{-4}\, \omega \text{ (Gauss)}, \tag{5.12}$$

$$|v_1| \cong 2.41 \times 10^{12}\, |E|/\omega \text{ (cm/sec)}, \tag{5.13}$$

$$|v_2| \cong 1.61 \times 10^{12}\, |E|/\omega \text{ (cm/sec)}, \tag{5.14}$$

$$|v_-| \cong 4 \times 10^{11}\, |E|/\omega \text{ (cm/sec)}, \tag{5.15}$$

$$\begin{aligned} r_1 &= |v_1|/\omega \text{ (cm)}, \\ r_2 &= |v_2|/\omega \text{ (cm)}, \\ r_- &= |v_-|/\omega \text{ (cm)}. \end{aligned} \tag{5.16}$$

As Eqs. (5.13) to (5.15) show, the macroscopic particle speeds have one and the same order of magnitude. The macroscopic electron speed is even smaller than the macroscopic ion speeds. In spite of the small mass ratios between electrons and ions, the electrons must not necessarily have a relativistic macroscopic speed, if the macroscopic ion speeds are almost relativistic. This point is very important.

According to Eq. (5.16), the radii of the translation circles have one and the same order of magnitude, too.

With appropriate numerical values for the quantities ω, $|B_0|$, and $|E|$ in Eqs. (5.12) to (5.15), non-relativistic macroscopic particle speeds with an order of magnitude of 10^8 cm/sec are possible. Then the relative speed between the two interpenetrating macroscopic D-ion and T-ion streams has an order of magnitude of 10^8 cm/sec, too. The latter value corresponds to a relative kinetic energy between these streams of about 100 keV per D-T-ion-pair. The optimal fusion impact cross section of the reaction $T(D,n)He^4$ 17.577 MeV has a value of about 5 barns and is located at a relative kinetic energy between the D- and T-ion of about 100 keV [6]. The specialized plasma is under electromagnetically generated fusion conditions, with $n_- = 10^{15}\,\text{cm}^{-3}$ and with a possible temperature of $T \geqq 10^5\,^\circ\text{K}$, far below the temperature of a thermonuclear D-T-plasma, which has to be about $10^8\,^\circ\text{K}$.

It is conceivable – whatever the probability is –, that natural plasma regions under quasi-stationary non-cut-off and "electromagnetic" fusion conditions exist in the universe. They must be large against the Debye length, the wave length, and the essential cyclotron radii. All applicability criterions of the considered solution have to be fulfilled. And the term "quasi-stationary" could mean that the plasma parameters change adiabatically and that the plasma is continuously regenerated by a possible total plasma stream with $v_0 \ll c$.

The generation of "electromagnetic" fusion conditions in laboratory plasmas is essentially limited by the possible values of ω (small wave lengths and small cyclotron radii are necessary) that require very high $|E|$ and $|B_0|$ values. This limitation is of technical nature.

[6] D. J. Rose and M. Clark, Plasmas and Controlled Fusion, M.I.T. Press, M.I.T., 1961.